Hydrodynamic Description of Transport Properties in Quantum Wires

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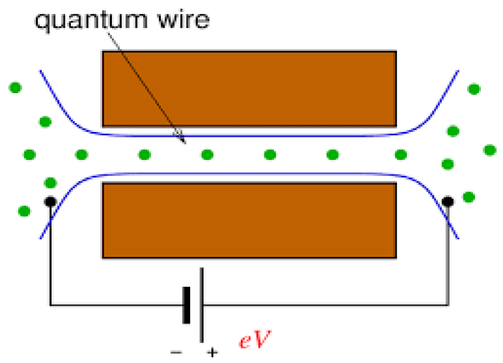
Introduction

Recently, there has been intense interest in the application of hydrodynamics to quantum gases and liquids [1]. However, standard hydrodynamics is not applicable to systems which have not fully equilibrated. For one-dimensional quantum liquids, electron backscattering can be exponentially slow. These systems can be described by two-fluid hydrodynamics, the same theory used to describe superfluid He-4 [2]. In the present context, the two-fluid model describes the electrons (the superfluid) and the gas of excitations (the normal component).

To date, our theoretical efforts have focused on connecting the hydrodynamic coefficients to transport phenomena. Quantities of interest include the thermopower, the Peltier coefficient, and the electrical resistance. Thus, our results have important implications for experiments on quantum wires.

We consider a 1D quantum wire in which both leads are at the same temperature. In order to maintain a current, the right-hand side of the wire is held at a higher electrochemical potential than the left side where both sides are connected to reservoirs. This causes the left movers to carry a different potential than the right movers, both moving away from their respective reservoirs.



Transport Properties

Consider a Galiliean invariant electron liquid at rest. Applying a temperature gradient, we find that

$$j_E = -\kappa \partial x T$$

The energy current *jE* may be written in terms of *u,* the velocity of the gas of excitations, with $j_E = T s_o n u$

$$\partial x T = -\frac{T s_o n}{\kappa} u$$

For a fluid moving with a velocity

$$v_d = \frac{I}{en},$$

we can go to the rest frame of the fluid by applying a boost. Thus,

$$\partial x T = -\frac{T s_o n}{\kappa}(u - v_d)$$

For In this case, the Peltier coefficient is

$$\prod = \frac{j_E}{I} = \frac{T}{e}\frac{\int \frac{s_o}{\kappa} dx}{\int \frac{1}{\kappa} dx}$$

Expressing the temperature gradient in terms of the energy and electric current, we have

$$\partial x T = -\frac{T}{\kappa e}\left(\frac{\int \frac{s_o}{\kappa} dx - s_o \int \frac{1}{\kappa} dx}{\int \frac{1}{\kappa} dx}\right) I$$

The power dissipated in the liquid is given by

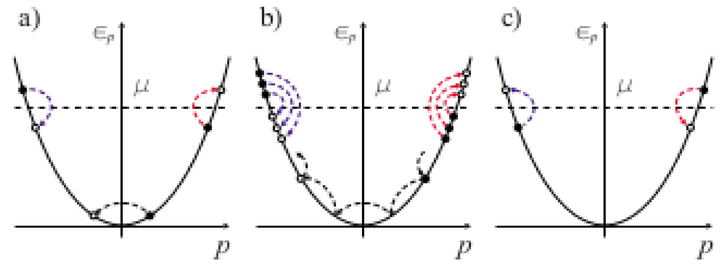
$$W = \left\langle \frac{\kappa}{T}(\partial x T)^2 + \zeta_2 (\partial x u)^2 + \zeta_3 \left[\partial x (v_d - u)\right]^2 + 2\zeta_1 \partial x u \partial x (v_d - u) \right\rangle$$

Now, we must express the velocities in terms of their currents and plug in our expression for the temperature gradient. Doing this, our expression for the resistivity becomes

$$\rho = \frac{1}{e^2}\left\langle \frac{T}{\kappa}(\delta s_o)^2 + \frac{e^2 \prod^2}{T^2}(\zeta_2 + \zeta_3 - 2\zeta_1)\left(\partial \frac{1}{s_o n}\right)^2 + \frac{2e\prod}{T}(\zeta_1 - \zeta_3)\left(\partial x \frac{1}{n}\right)\left(\partial x \frac{1}{s_o n}\right) + \zeta_3 \left(\partial \frac{1}{n}\right)^2 \right\rangle$$

Outlook

We have seen the usefulness of the two-fluid hydrodynamic description when applied to systems of this nature. Moving on from this, our goal is to next calculate the same properties in the microscopic context. We can examine the processes at play in the electron fluid in order to confirm that we can derive a similar form of this result from another starting point. The microscopic form should result in a description which involves three particle collisions that change the number of right movers in the system. This process involves the scattering of two electrons at the Fermi level plus one electron at the bottom of the band. This is the most important factor in altering the current, as shown below in a). [3]



Conclusions

We have explored the transport properties in one-dimensional quantum wires, with a focus on the Peltier coefficient and resistivity. These quantities will have applications to experiments of this nature.

References

[1] Andreev A.~V., Kivelson S.~A., Spivak B., 2011, PhRvL, 106, 256804.
[2] DeGottardi W., Matveev K.~A., 2020, PhRvL, 125, 076601.
[3]Levchenko A., Micklitz T., Rech J., Matveev K.~A., 2010, PhRvB, 82, 115413.