Quantum algorithms for mathematical optimization

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Summary

- Current activity in QIC
- What is mathematical optimization?
- Quantum algorithms for mathematical optimization
- A new approach for quantum optimization

Current research in QIC

- Quantum search algorithms, with Rafael Cação and LNCC
- Quantum gradient, with Lucas Cortez and Macquarie University
- Quantum walk, with LNCC

Current research on QIC

- Quantum computational chemistry, with Prof. Jorge Morales
- QIC education, with Profs. Beth Thacker, Yuanlin Zhang, and Jianlan Wang

On the top of that… QIC

- graduate classes
- seminar series

What is an optimization problem?

In short, it is the problem of finding the maximum or minimum of a function $f : D \rightarrow \Re$ over a subset S of its domain, i.e.

$$\text{maximize } f(x)$$
$$\text{s.t. } x \in S$$

Are optimization problems relevant?

"Nothing in all the world will occur in which no maximum or minimum rule is somehow shining forth" *Leonhard Euler*

Optimization problems appear in

- Manufacturing
- Aviation
- Sports
- Medical sciences
- Finance
- Telecommunications
- Mathematics, physics, chemistry, biology, computer science, etc.

QIC in industry

Emphasis on:

- NISQ approach to computational chemistry
- Quantum algorithms for solving mathematical optimization application models

Quantum algorithms for mathematical optimization

- Grover search for finding the maximum
- QAOA and the adiabatic approach
- Quantum algorithms for:
  - Gradient descent
  - Linear programming

Computational complexity

- For the most part, *convex* optimization problems are solvable in polytime
- The vast majority of optimization models arising in real-life applications are NP-hard
- The state-of-the-art consists in solving such models by solving a sequence of convex relaxations… and here's where QIC makes a difference

A new approach

Quantum walk to speed up convex optimization problems. Specifically:

- Use the quantum walk to speed up mixing in the Markov chain of the problem
- Perform sampling and search to obtain approximate optima with high probability.

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