Investigating Graduate Students' Knowledge and Comprehension of Lagrangian Mechanics: Rubric Development

Kyle Wipfli, Beth Thacker

Texas Tech University

Abstract

Over the years many different modalities have been created and studied to improve the physics classroom at the K-12 and undergraduate level. However there has been very little to no published research on how graduate students learn with these different modalities that improve the physics classroom. We have created materials to teach graduate level methods to solve classical systems through Lagrangian mechanics and analyze the symmetries of the system. To analyze these materials, we are developing a rubric for analyzing answers to Lagrangian mechanics problems. We are using the rubric to analyze both an inquiry-based modality and a lecture based modality. This poster will go over the development of the materials and the application of the rubric.

Classroom Environment and Data Collection

In this section we will go over the different contexts in which students learned graduate level classical mechanics, and the data collection procedure.

Inquiry Based Method

The inquiry-based method to teach students Lagrangian mechanics was a blended method between structured inquiry and open-ended inquiry. Structured inquiry is an approach where a set of materials guides students though a sequence of steps or methods to learn a specific topic or concept. Open-ended inquiry is an approach where students explore and solve problems on their own, while using the teacher as a helping guide when needed.

In Fall 2022, students learned classical mechanics through an open-ended inquiry course with the occasional lecture to explain certain material. This course was based on using Mechanics: Course of Theoretical Physics Volume 1 by Landau and Lifshitz. Materials were developed separately to teach how to set up and solve for the equations of motion using lagranian formalism based on structured inquiry methods. These materials were then given to the class to use to learn how to set up Lagrangian equations, how to solve for equations of motion, and how to get conservation laws/symmetries by using the Euler-Lagrange equations.

Approximately 20 students took a pretest where they answered a question about a constrained particle and told to use Lagrangian methods and identify the symmetry (problem given blow.) For a post-test, students' final exams were analyzed based on 5 Lagrangian mechanics problems.

Traditional Method

The traditional method uses lecturing introduce and teach classical mechanics. The textbook used for this class is Classical Mechanics by Goldstein. We used the same test as the inquiry-based pretest for this class because we were only able to get a post test for the traditional class.

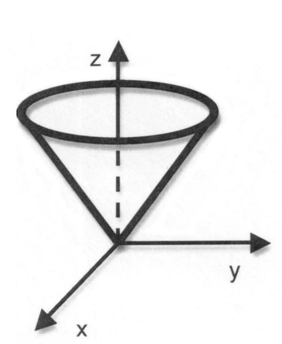
Test Given

A particle of mass m is constrained to move under gravity without friction on the inside surface of a smooth cone, i.e., z (r ,θ)=a*r ,where a is a constant (see Figure).

a) Find the equations of motion.

b) What are the conserved quantities? Explain why.

c) What transformation(s) is the motion invariant under?



Introduction

Over the past 40-50 years, the physics education research community has investigated improving understanding and learning methods in the K-12 and undergraduate physics classrooms. Today we have found that inquiry-based methods and active engagement play a key role into improving the learning outcomes in the physics classroom.(1,2) However, there have been some studies of how student comprehension and learning methods affect the graduate level physics classroom.(3,4,5) Yet there has been little to no studies in how they affect the graduate level classical mechanics classrooms. The aim of this research project is to compare inquiry-based methods and traditional lecture methods when learning Lagrangian mechanics in a graduate level physics course. To do so, we have developed structured inquiry-based learning materials to analyze and a rubric to analyze student knowledge and comprehension with. In this poster we introduce the rubric, the test which is given analyzed, the development of the learning materials and rubric, and future to validate, test reliability and implement the rubric to gauge student comprehension and knowledge.

Lagrangian Mechanics Rubric

Evidence of:	Points	ex: 2D	Physical vs Mathematical Sensemaking	Learning Goal Addressed	Examples of Getting full Points
Correct Kinetic Energy	1	1	Physical	Students show the basic knowledge of the Lagrangian formalism	Write down the correct expression for T: T=½(dx/dt^2) in 1D
Correct Potential Energy	1	1	Physical	Students show the basic knowledge of the Lagrangian formalism	Write down the correct expression for U: U=mgy for gravity acting in y-direction
Correct constraint application	1	1	Mathematical	Students show knowledge of mathematical processes when using Lagrangian formalism	For a person who is constantly at 10m in the z-direction. Plugging in z=10m and dz/dt =0m/s
Identification of Symmetry from eq	1	1	Blended	Students show knowledge of symmetries in classical mechanics	Write down the correct symmetry/conservation law due to evaluating the E-L equations
Identification of Symmetry from coordinates	1	1	Physical	Students show knowledge of symmetries in classical mechanics	Write down the correct symmetry/conservation law due to ignorable coordinates . I.e. L=/= L(θ) Therefore angular momentum in the θ direction is conserved
Correct eq of motion(s)	Number of eq's	2	Blended	Students show knowledge of mathematical processes when using Lagrangian formalism	Students must be able evaluate the E-L equations and get N equations for an N-D system
				Students show knowledge of symmetries in classical mechanics	
Correct coordinate transformations	1	1	Physical	Students show the basic knowledge of the Lagragian formalism	Students must be able to choose the correct coordinate system to create a
Simplifying equations of motion	Number of eq's	2	Mathematical	Students show knowledge of mathematical processes when using Lagrangian formalism	
	Total number of points	10			

Future Statistical Analysis for Reliability and Validity of the Rubric

The rubric has been created but still needs to be tested for reliability and validity. Reliability of rubrics are scored through two measures: intra-rater reliability and inter-rater reliability. These are typically found by calculating Cohen's Kappa or Gwet's AC1. We plan on using both measures of reliability to statistically test the reliability of the rubric, once emergent coding process has finished.

They both use the equation $\frac{p_o - p_e}{1 - p_e}$ (6) for the statistic.

Where p_o is the percent chance of agreement and p_e is the expected chance of random agreement.

The difference is that p_e is calculated differently.

The reason for both measures is that Cohen's Kappa is a historically accepted measure for reliability between two raters, while Gwet's AC1 has an easier interpretation of the scores.

Validity of rubrics is assessed through multiple measures also. The measures relevant to this rubric are construct validity, content validity, and criterion validity. Construct validity ensures the tool measures the construct under question. Construct validity is achieved by proper creation of a rubric through standardized coding schemes, such as emergent coding. Content validity ensures a rubric has all content covered for the purpose of evaluation. This is achieved through applying and revising a rubric to evaluate all skills shown. Criterion validity ensures a tool has the power to discern between different responses and thus its ability to predict future responses. This rubric will be evaluated based on item difficulty (δ_i) and person's ability (β_n) from the Rasch Model which is given by:

$$\Pr\{X_{ni} = 1\} = \frac{e^{\beta_n - \delta_i}}{1 + e^{\beta_n - \delta_i}},$$

(7)

Materials and Rubric Development

The aim of this project is to compare students' knowledge and comprehension of Lagrangian mechanics in graduate level classical mechanics between inquiry-based teaching methods and traditional lecture teaching methods. To accomplish this we have developed a set of learning materials and a rubric.

The learning materials have two major sections. The first section goes over Lagrangian mechanics and how to solve the problems using the Euler-Lagrange equations and the method of Lagrange multipliers. This section starts with defining the differential operator, then uses the Principle of Extremal action and Newton's Second Law to define the Lagrangian as a function that satisfies both theories. The materials then go over how to set up a Lagragian, choose correct coordinate systems, and use method of Lagrange multipliers. After each topic is a checkpoint where a teacher then asks students questions to gauge if the students know the material or not. This section wraps up with analysis of conservation laws, ignorable coordinates and the symmetries of translational/rotational invariance. These materials are the bulk of the knowledge base that is being tested over. The second section is over group theory and the connection to physical transformations. This is an optional part of the material and has only been tested once.

The rubric is used to test students' knowledge over Lagrangian mechanics and symmetries. The rubric specifically looks at what type of sensemaking students can use and what specific parts of solving a Lagranian mechanics problem signifies the different types of sensemaking. We used the process of emergent coding to develop this rubric. Emergent coding is where you take responses and categorize different elements of the responses based on what has been seen. This is a continual process, and the rubric is thus able to change with each new data set, as to refine the categories (called codes), and the way they are scored. Thus far we have only used data from the inquiry-based class and will continue the emergent coding process with the traditional lecture-based class.

Conclusion

Thus far we have created a rubric for evaluating different skill sets based on Lagrangian mechanics and identification of symmetries based on emergent coding. We have collected data for two classes and plan to use statistical analyses to verify independence, use Rasch analysis see the difference between the different populations. In the future we plan on report on the relative effectiveness between inquiry-based approach and traditional approach.

References & Acknowledgements

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Acknowledgement Funding: American Physical Society Forums on Education (APS FEd)